Managing Risk in Pension Plans

Following the withdrawal of sponsors, traditional defined benefit (DB) systems for retirement provision are rapidly being replaced by hybrid or defined contribution (DC) systems, in which future retirement income is less certain.1 The long-lasting turmoil in the financial markets since 2008 emphasizes the challenges that pension funds face in developing strategies which focus on “delivering an adequate target pension with a high degree of probability” (Blake, Cairns, and Dowd 2008; Merton 2010). In the study described here, we investigate whether the use of equity- and volatility-based derivative instruments may help the pension industry in meeting such challenges by considering the design problem for optimal derivative strategies in the context of strategic asset allocation. To do so, we use asset models that incorporate risks such as stochastic volatility and the possibility of sudden price jumps. To get a clear interpretation of the end results, we use a stylized expected utility framework to capture the preference of pension funds. By incorporating realistic performance-evaluation criteria of pension investors and by assuming that trading cannot be performed continuously, we can draw conclusions that are relevant in a practical setting.

Traditional asset allocation focuses on diversification across asset classes. We propose an allocation strategy that emphasizes diversification across risk premia (i.e., across underlying risk factors) instead. It is increasingly recognized that return variance on financial securities is stochastic and that sudden jumps in asset prices may appear. Variance uncertainty and price jumps have therefore been treated as important additional risk factors in the recent scientific literature,2 and there are significant risk premia associated with both (Carr and Wu 2009; Bollerslev, Gibson, and Zhou 2011). The extra volatility and jump risks generate market incompleteness and hence make derivatives non-redundant.3 Including derivatives may thus enhance investment performance, since it allows investors to diversify across risk factors and the associated risk premia.

The possibility of earning volatility and jump risk premia may be particularly interesting for investors with a long time horizon. Focusing on the long investment horizon, Litterman (2011) argues that pension funds may be more capable of bearing volatility and jump risks than other financial institutions, since volatility is mean-reverting over longer time horizons and jumps will hurt the objectives of short-term investors much more than those of long-term investors. Acknowledging this long-term argument, we also consider the downside risk constraints of pension funds at short-term horizons. In periods when the liquidity of certain assets is reduced, derivative contracts can become valuable instruments to generate future

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The existence of stochastic jumps in asset dynamics and the absence of trading possibilities over a certain time horizon lead to similar forms of market incompleteness, which derivatives may help to mitigate. Moreover, derivatives make it possible to implement an extensive rebalancing strategy for certain assets without trading in these assets themselves.

Key Findings

Using an extended asset menu with equity and variance derivatives, we find that the optimal portfolio not only markedly improves welfare in the expected utility framework but also improves on most (and in some cases all) other evaluation criteria frequently used by pension funds. If downside risk is constrained to be no worse than the benchmark case (stocks and bonds only), the suboptimal portfolio with derivatives can still outperform the benchmark case on all evaluation criteria.

The portfolio optimally loads on equity risk premium, volatility risk premium, and jump risk premium by holding a long position in equity and a short position in variance derivatives. It also contains a long position in the out-of-the-money (OTM) put and a short position in the OTM call, which resembles a so-called collar strategy. The portfolio loads on volatility risk via variance derivatives because these give investors a controlled exposure to this risk, whereas options’ sensitivity to volatility risk depends on the remaining time to maturity and the stock price path. When variance derivatives are present in the portfolio, one can further enhance retirement income security by a short position in calls and a long position in puts, but not necessarily in equal amounts.

The use of shorted calls to help pay for the purchase of puts is based on the intuition that to improve the chances of achieving a desired income target in pension plans, upside potential must be relinquished if no extra external funding is available. Timmermans, Schumacher, and Ponds (2011) propose a synthetic collar position (long OTM puts and short OTM calls in equal amounts) to enhance income security for a given predefined level of downside risk. Our results indicate that this is still possible when the uncertainty in asset prices varies with time and when asset prices may be discontinuous, as long as volatility-based derivatives can be used in the asset allocation.

In the next section we explain the model, the asset menu, and the optimization framework. We then present our main results and perform robustness checks. The article concludes with a discussion of some practical implications and extensions.

Modeling Setup

The Economy

We use a realistic asset model that includes stochastic volatility and price jump risk, as in Liu and Pan (2003). According to Broadie, Chernov, and Johannes (2009), such a model does incorporate the major factors driving equity option returns.

We assume that the economy contains a riskless asset that earns a constant rate of return $r$ per year and a risky asset (representing a diversified equity index) that is subject to both continuous price changes by diffusion and discontinuous price shocks. The stochastic volatility process is the one introduced by Heston (1993) and features mean reversion of the equity variance process $V$ to a long-term equilibrium level of variance $v$. The speed of mean reversion is controlled by the parameter $\kappa$, and the uncertainty in the volatility is controlled by the parameter $\omega$, the “volatility of volatility.” The asset dynamics are given by the following stochastic differential equations:

$$
\begin{align*}
    dB_t &= rB_t dt \\
    dS_t &= \left(r + \rho \right) S_t dt + S_t \sqrt{V_t} dY_t + \mu S_t \left( dN_t - \lambda V_t dt \right) \\
    dV_t &= \kappa \left( \bar{V} - V_t \right) dt + \omega \sqrt{V_t} \left( \rho dY_t + \sqrt{1 - \rho^2} dZ_t \right)
\end{align*}
$$

where $B$ and $S$ are the price processes for the riskless and risky assets (“Bonds” and “Stocks”). The stochastic processes driving the dynamics are the standard Brownian motions $Y$ and $Z$ and the pure jump process $N$, which are all assumed to be independent. The correlation coefficient $\rho$ between the price process $S$ and the variance process $V$ is assumed to be negative.

The jump process $N$ is assumed to have a jump intensity that is proportional to the variance process, as in Liu and Pan (2003), with proportionality constant $\lambda > 0$. This implements the empirical observation that in times of increased market uncertainty, the probability of a relatively large price change in the risky asset increases. The stochastic jump size $\mu$ is assumed to be fixed and downward, so we take $-1 < \mu \leq 0$.

The equity risk premium $p_t$ compensates for diffusive risk and jump risk: it takes the form $p_t = \eta V_t + \mu \left( \lambda - \lambda^0 \right) V_t$, where $\eta$ is the market price of diffusive risk and $\lambda^0$, the risk-neutral analogue of $\lambda$, controls the reward for jump risk. The variance risk premium is controlled by $\zeta$, the market price of volatility risk, which enters through the stochastic pricing kernel:

$$
\begin{align*}
    \frac{d\pi_t}{\pi_t} &= -rdt - \sqrt{V_t} \left( \eta dY_t + \zeta dZ_t \right) \left( \lambda^0 / \lambda - 1 \right) dN_t.
\end{align*}
$$

One can show that the implied variance risk premium parameter in this model is $\omega (\eta + \zeta \sqrt{(1 - \rho^2)})$. Thus, the variance risk premium is decomposed into two parts: the first is due to the correlation with the equity diffusive risk, and second is
a compensation for the independent variance risk. Because of a negative correlation between the price process $S$ and the variance process $\nu$, and a negative value of the market price of variance risk $\zeta$, the resulting variance risk premium parameter is negative. Christoffersen, Jacobs, and Heston (2011), who consider a similar decomposition, show that the negative market price of volatility risk not only generates higher implied volatility than realized volatility but also causes a higher variance and a higher autocorrelation of implied volatility, which explains the fatter tail of the risk-neutral density relative to the realized return density.

**Derivative Instruments and Asset Menu**

Apart from the basic underlying stocks and bonds, we consider different possibilities for the derivatives that can be added to the portfolio. From a hedging perspective, OTM puts can, for example, be used to compensate for decreasing stock prices and, in particular, as protection against the effects of a downward jump in the risky asset. The combination of an OTM put and a shorted OTM call, mentioned above, may be used to transfer probability mass from the investment portfolio’s rates of return for relatively extreme economic scenarios (in which the stock price increases or decreases dramatically) to more moderate ones.

Since the stock process used in our economic scenario generator exhibits stochastic volatility, the return on the options that we consider depends not only on the movement of stock prices but also on the dynamics of volatility. This is the motivation to analyze the effect of adding another derivative to the asset menu, one that has a stable and pure exposure to volatility. We therefore introduce a variance product (the floating leg of a so-called variance swap) that generates a payoff that depends on the realized variance of the risky asset during a prescribed period (in this case, the investment horizon). As such, it is linked only to the stochastic volatility process and not directly to the asset price, although there is an indirect exposure due to the correlation between the stock price and its volatility. Introducing a direct volatility-linked derivative will help us to distinguish between volatility risk premia and jump risk premia, and thus overcome the difficulty in making this distinction reported by Pan (2002), who used only call and put options.

**Investor Preferences and Expected Utility Framework**

We consider the optimization of expected utility as a way to generate portfolios that consistently trade off risk and returns, but such portfolios must then be analyzed in terms of the more intuitive performance criteria that pension fund managers are more familiar with. Our analysis therefore examines both.

The expected utility framework is commonly used to define an objective performance measure for the optimal portfolio choice problem. For tractability and consistency with the existing literature, we take the same approach here. To specify the risk preferences of the pension fund, we consider a utility function with constant relative risk aversion parameter $\gamma$. Furthermore, to be close to the pension investment practice under a solvency constraint, we use a displaced version of this constant relative risk aversion (CRRA) utility function with a threshold value that guarantees that the fund can never lose more than a percentage $h$ (which we take to be 50%) of its value per investment period. The fact that we choose a threshold on the return over a period instead of a fixed level of wealth for all periods can be interpreted as the implementation of a “habit formation” effect. We thus formulate our optimization problem as

$$\max_{\alpha} E_t \left[ \frac{(W_t - hW_t)^{1-\gamma}}{1-\gamma} \right]$$

with $W_t$ the wealth at the initial time $t$ and $W_t = W_0 \sum \alpha_i A_i$, where $A_i$ denotes the value at time $t$ of the $i$th asset in the asset menu and $\alpha_i$ is the associated portfolio weight, which is constant per investment period. The asset menu includes equity, bonds, equity puts and calls, and the variance product.

**Real-Life Evaluation Criteria**

Although the expected utility framework is useful to determine an optimal investment strategy using a well-defined single-goal function, additional criteria must be considered as well to draw conclusions that are relevant to pension funds. We assume that the fund starts from a position of holding assets and liabilities that have the same value – that is, the initial funding ratio equals 100%. We consider several characteristics of the fund after an investment period of one year, the investment planning horizon for most funds. First we will look at the 2.5% quantile of the funding ratio and the expected shortfall at that level, also known as the value at risk (VaR) and tail value at risk (TVaR) respectively. These measure the maximal and average level of the funding ratio in the 2.5% worst economic scenarios that are generated in our simulation.

We also report the standard deviation of the rate of return as a general measure of (both upward and downward) risk. These risk measures are compared with the main statistics, which consider averages over all scenarios, the mean rate of return, and the median rate of return. Lastly, we consider measures of income security, represented by the probability of reaching specific return targets. For example, we report the probability of a realized rate of return that is larger than the risk-free rate and the probability of a rate of return that is larger than the risk-free rate plus the inflation rate (taken as 2% annually throughout this article). The risk-free rate plus 2% thus represents the investment target of pension fund participants in the study.
When Do Derivatives Add Value in Pension Fund Asset Allocation?

Parameter Calibration and Solution Method

To determine a plausible value for the risk aversion parameter $\gamma$ for a typical pension fund, we calibrate it to obtain plausible stock holdings for the optimal portfolio problem without derivatives, without jump risk, and without stochasticity in the volatility. In other words, we determine the optimal percentage to invest in stock as a function of the parameter $\gamma$ and then choose this value to obtain a stock investment of 45%, which roughly corresponds to the equity holdings of a typical Dutch pension fund.

Parameter values are as indicated in Table 1 (the base case) unless stated otherwise. These parameters are chosen within the range reported in the extensive empirical literature for this model. In particular, the values are based on or close to those reported by Pan (2002); Liu and Pan (2003); Liu, Pan, and Wang (2005); and Broadie et al. (2007, 2009). Under these parameter values, the equity risk premium is 9.18%, which is a combination of jump risk premium (3.67%) and diffusion risk premium (5.51%). The risk premium for variance risk is $-3\%$ ($= v\zeta$), and jumps in stock prices of $-10\%$ occur at an average intensity of $0.36$ per year.

Santa-Clara and Yan (2010) estimate the ex ante perceived severity of risks and the associated risk premia. They find that the ex ante expected jump size is $-9.8\%$, a compensation for stochastic volatility risk of $5\%$ and jump risk of $6.9\%$. In a closely related study, using a different sample period and estimation method, Pan (2002) identifies a jump premium of $3.5\%$ and a volatility premium of $5.5\%$. Using the model-free realized volatility (measured from high-frequency equity prices) and implied volatility (measured from option prices), Bollerslev, Gibson and Zhou (2011) extract the volatility risk premium parameter (i.e., $\kappa^Q - \kappa$), which turns out to be highly time-varying, with an average value of about $-1.8$. In our baseline parameter setting, we take a more conservative $\kappa^Q - \kappa = -1.1$.

As mentioned in the studies cited above, estimating these risk premia precisely is very difficult, so our choices represent a trade-off between different reported values. However, the results of sensitivity tests shown later in the article indicate that all our conclusions remain valid under fairly broad parameter ranges and even when the volatility risk premium or the jump risk premium is set to zero.

Table 1: Parameter Values (Base Case)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion</td>
<td>$\gamma = 5.50$</td>
</tr>
<tr>
<td>Habit-formation parameter</td>
<td>$h = 0.50$</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r = 0.030$</td>
</tr>
<tr>
<td>Mean reversion speed</td>
<td>$\kappa = 6.40$</td>
</tr>
<tr>
<td>Equity jump intensity</td>
<td>$\lambda = 24.00, \lambda^Q = 48.00$</td>
</tr>
<tr>
<td>Equity jump size</td>
<td>$\mu = -0.100$</td>
</tr>
<tr>
<td>Volatility of volatility</td>
<td>$\omega = 0.30$</td>
</tr>
<tr>
<td>Equilibrium variance</td>
<td>$\nu = 0.0153$</td>
</tr>
<tr>
<td>Initial volatility</td>
<td>$\sqrt{(V_0)} = 0.124$</td>
</tr>
<tr>
<td>Compensation equity risk</td>
<td>$\eta = 3.60$</td>
</tr>
<tr>
<td>Compensation volatility risk</td>
<td>$\zeta = -2$</td>
</tr>
<tr>
<td>Correlation between diffusive equity risk and variance risk</td>
<td>$\rho = -0.53$</td>
</tr>
</tbody>
</table>

We approximated the distribution of the asset returns using a simulation for an investment horizon of one year that involved 250,000 paths with 1,200 time steps per simulation each. We used numerical optimization to find the optimal portfolio in terms of expected utility of final wealth, and exact equations to check local first-order optimality conditions.

Results

Allocation with Stocks and Bonds Only

Table 2 summarizes our main results, and Figure 1 summarizes the return distributions of the optimal portfolios. For the base case without derivatives, we calibrate the risk aversion parameter to achieve an equity investment corresponding to 44% of the initial wealth and generating a certainty equivalent rate of return (CER) of 5.28%. This portfolio reflects the equity investment of a typical pension fund, which represents a realistic benchmark for our study.

The VaR and TVaR values of $-5.22\%$ and $-7.79\%$; the indicated mean, median, and standard deviation of the rate of return ($7.33\%, 7.42\%$, and $6.28\%$ respectively); and the probabilities of earning more than the risk-free rate or even more than the risk-free rate plus inflation ($76\%$ and $65\%$ respectively) reported in line 1 of Table 2 form the baseline to which all subsequent results should be compared.
In the second strategy, we include an OTM put option in the asset mix, with a maturity of 15 months and a strike equal to 95% of the initial equity value. It is interesting to note that instead of directly protecting the stock investment using a long position in the put, it is better to reduce stock holdings and sell puts (i.e., a short position in OTM puts of −0.79%). This confirms the results reported by Driessen and Maenhout (2007) that under CRRA utility, puts often turn out to be too expensive for the protection they offer in downward scenarios. The notional amount of the puts is about 57% of the stocks invested. In this strategy, risk (measured in terms of SD or VaR) is reduced by a smaller investment in stocks combined with a shorted put option, instead of by a long position in puts.

The reason for this is the negative volatility premium, which makes it attractive to have a net short position in derivatives. The overall effect of this combination on the distribution of the funding ratio at the end of the investment horizon is a slight improvement in VaR and a substantial decrease in SD, with relatively little change to the mean and median of the rate of return. The probabilities of earning the risk-free rate and of earning the risk-free rate plus inflation both increase by a few percentage points. Although VaR improves, TVaR becomes slightly worse because of the payoff needed for the shorted put under scenarios in which the stock price decreases substantially. This effect nicely illustrates the limitations of looking only at the 2.5% quantile as a risk measure, without considering behavior in the tail of the distribution.

The attractiveness of shorting derivatives rather than taking long positions is also underlined in a strategy in which we take the same OTM put options together with OTM call options with the same maturity and a strike at 115% of the initial equity value. This strategy contains a short call (corresponding to 60% of stocks) and a short put (corresponding to 8% of stocks) and roughly equal weights in stocks and bonds. Compared to the optimal portfolio, which contains only puts, the probabilities of achieving the risk-free rates with and without inflation increase 0.3% and 3% respectively, but VaR and TVaR worsen slightly; the mean is the same, but the median improves by 0.7% and the SD is 0.3% lower, indicating that the distribution has not only become more compact but is also “tilting” more toward positive values.

### Allocation including Variance Derivatives

We see a more distinct example of this tilting effect in the fourth strategy, in which the variance product is added to the asset menu. When shorted, it gives direct exposure to variance risk and earns the associated volatility premium. This portfolio is welfare enhancing, with CER up to 5.89% (from 5.28% in the benchmark case), because investors can make explicit trade-offs and diversify between diffusion risk, jump risk, and volatility risk, earning the compensating risk premia. We see that stock holdings are reduced dramatically (compared to the first strategy, which uses only stocks and bonds), as the shorted variance product has a positive correlation with stock. As in the second strategy, VaR improves while TVaR deteriorates a bit, but this time we see a spectacular improvement in mean and median of the rate of return (up 0.5% and 1.2% respectively), while SD is also reduced by 0.8%. The probabilities of earning more than the risk-free rate and earning more than the risk-free rate plus inflation improve markedly, by 6% and 8% respectively.

### Table 2: Optimal Portfolios under the Default Parameter Setting

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Bonds</th>
<th>Stocks</th>
<th>Put</th>
<th>Collar</th>
<th>Variance</th>
<th>Call</th>
<th>CER</th>
<th>VaR</th>
<th>TVaR</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Prob R</th>
<th>Prob Infl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.91%</td>
<td>44.09%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.28%</td>
<td>−5.22%</td>
<td>−7.79%</td>
<td>7.3%</td>
<td>7.4%</td>
<td>6.3%</td>
<td>76%</td>
<td>65%</td>
</tr>
<tr>
<td>2</td>
<td>64.75%</td>
<td>36.04%</td>
<td>−0.79%</td>
<td></td>
<td></td>
<td></td>
<td>5.32%</td>
<td>−5.06%</td>
<td>−8.36%</td>
<td>7.1%</td>
<td>7.4%</td>
<td>5.6%</td>
<td>80%</td>
<td>68%</td>
</tr>
<tr>
<td>3</td>
<td>56.83%</td>
<td>43.18%</td>
<td>−0.01%</td>
<td></td>
<td></td>
<td></td>
<td>5.28%</td>
<td>−5.20%</td>
<td>−7.80%</td>
<td>7.3%</td>
<td>7.4%</td>
<td>6.3%</td>
<td>76%</td>
<td>65%</td>
</tr>
<tr>
<td>4</td>
<td>84.60%</td>
<td>23.67%</td>
<td>−8.27%</td>
<td></td>
<td></td>
<td></td>
<td>5.89%</td>
<td>−4.96%</td>
<td>−8.37%</td>
<td>7.8%</td>
<td>8.7%</td>
<td>5.5%</td>
<td>82%</td>
<td>73%</td>
</tr>
<tr>
<td>5</td>
<td>51.94%</td>
<td>49.01%</td>
<td>−0.12%</td>
<td></td>
<td></td>
<td></td>
<td>5.34%</td>
<td>−5.50%</td>
<td>−8.54%</td>
<td>7.1%</td>
<td>8.1%</td>
<td>5.3%</td>
<td>80%</td>
<td>71%</td>
</tr>
<tr>
<td>6</td>
<td>74.92%</td>
<td>33.59%</td>
<td>0.74%</td>
<td></td>
<td></td>
<td></td>
<td>5.91%</td>
<td>−5.45%</td>
<td>−8.64%</td>
<td>8.0%</td>
<td>9.1%</td>
<td>5.7%</td>
<td>81%</td>
<td>73%</td>
</tr>
</tbody>
</table>

CER = certainty equivalent rate of return; VaR = value at risk; TVaR = tail value at risk; SD = standard deviation; Prob R = probability of earning more than the risk-free rate; Prob Infl = probability of earning more than the risk-free rate plus inflation.
Including both options and the variance product in the asset mix in the sixth and last strategy yields additional small improvements in the CER, mean, median, and the probability of earning more than the risk-free rate plus inflation. The optimal strategy includes a similar investment in the variance product, but the investment in equity increases from 23.7% to 33.6%, and this time we indeed get a construction resembling a collar. A long investment in puts (with 55% stock covered) and a short position in calls (31% of stock covered) together reduce the upside potential of the stock returns but protect against adverse scenarios. Since the variance product is part of the asset menu, the overall exposure to volatility risk will be such that investors can earn volatility risk premia despite having a long position in put options. We thus see a combination of volatility as a risk factor that we can invest in while still buying protection for our equity exposure.

**Summary of Study Findings**

First, we notice that in an environment that includes stochastic volatility with an associated negative risk premium, direct protection by put options is suboptimal. This is true even if the protection is financed by writing calls on relatively high stock prices. Instead, risk can be reduced by taking a smaller investment in stocks and writing put options. The effect is illustrated in Figure 2, where we show the rate of return of our portfolio as a function of the rate of return of the stock. We see that exposure to both advantageous and disadvantageous extreme scenarios has indeed been reduced, and that more probability mass of the distribution of the wealth will be shifted to the center as a result.
Second, once derivatives allow us to take a direct exposure to volatility without exposure to (the direction of) stock prices, the investment opportunity set is enlarged, and volatility can then be considered an asset class on its own. Investors thus can make trade-offs among distinct risk factors according to their preferences. If the variance product is shorted, it has a positive risk premium (driven by volatility risk and jump risk) and a positive correlation with stock returns, partly because of the direct correlation between volatilities and stock prices, but also because when stock prices jump downward, the variance product loses value as well; this effect makes VaR smaller but TVaR a bit higher relative to the situation in which no variance products have been added to the asset mix. However, a larger tail risk as measured by TVaR can be remedied, as we will show in the next section when we discuss robustness.

Controlling Tail Risk and Sensitivity Analysis

When we add a constraint that tail risk for our optimized portfolio should not exceed the value for the benchmark portfolio (stocks and bonds only), we find that higher income security and welfare enhancement can still be obtained using our extended asset menu. Table 3 shows an example, in which we take the best portfolio from the preceding section but reduce equity holdings by 5% and increase bond holdings by the same amount. The resulting portfolio now dominates the optimal stock and bond portfolio along all evaluation criteria, including the two tail risk indicators (TVaR and VaR). Of course, this portfolio is slightly suboptimal in terms of CER, which emphasizes that while optimizing expected utility of terminal wealth is a useful tool to generate candidate portfolios, it may be beneficial to tweak the results to improve along different performance criteria that are more important in practice.

Since derivatives are marked to market throughout the holding period, the daily movements in the profit and loss (P&L) accounts of the derivatives might be a concern for investors. To address this issue, we investigated the tail risks in the P&L of the variance product during the holding period in our model. In our simulations, the distribution of the P&L turns out to show rather limited downside risk, which can be explained by the relatively long maturity of the variance product.

Sensitivity Analysis

Although our discussion has been based on results for the specific parameter setting that we chose as our base case, the structure of the optimal solution, and therefore our conclusions, seem to be fairly robust in the face of parameter changes. Tables 4 and 5 push the sensitivity analysis to an extreme by assuming a zero risk premium for either jump risk (Table 4) or volatility risk (Table 5). In such an environment, the qualitative message still remains, although quantitatively reduced in size, demonstrating that welfare enhancement depends critically on investors’ beliefs about volatility and jump risk premia.

Table 3: An Adjusted Derivative Portfolio with Tail Risk No Larger Than the Benchmark Case

<table>
<thead>
<tr>
<th>Bonds</th>
<th>Stocks</th>
<th>Put</th>
<th>Collar</th>
<th>Variance</th>
<th>Call</th>
<th>CER</th>
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<th>SD</th>
<th>Prob R</th>
<th>Prob Infl</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.91%</td>
<td>44.09%</td>
<td></td>
<td></td>
<td></td>
<td>0.74%</td>
<td>-8.33%</td>
<td>-5.22%</td>
<td>-7.79%</td>
<td>7.3%</td>
<td>7.4%</td>
<td>6.3%</td>
<td>76%</td>
<td>65%</td>
</tr>
<tr>
<td>79.92%</td>
<td>28.59%</td>
<td>0.74%</td>
<td></td>
<td>-0.43%</td>
<td>5.87%</td>
<td>-4.66%</td>
<td>-7.63%</td>
<td>7.5%</td>
<td>8.6%</td>
<td>5.1%</td>
<td>82%</td>
<td>73%</td>
<td></td>
</tr>
</tbody>
</table>

CER = certainty equivalent rate of return; VaR = value at risk; TVaR = tail value at risk; SD = standard deviation; Prob R = probability of earning more than the risk-free rate; Prob Infl = probability of earning more than the risk-free rate plus inflation.
**Longer-Horizon and “Rolling Over” Strategies**

Since pension funds have a long-term investment horizon, we extend the horizon to four years and investigate two cases. In the first case, both the investment horizon and the time to maturity of the derivatives are extended, so that the investor can follow the same buy-and-hold strategy over the longer horizon as in the baseline case. Table 6 shows that the portfolio including variance derivatives improves performance along all criteria, including VaR and TVaR, over a four-year period. The optimal portfolio has a relatively large shorted position in variance product.

Since equity and variance derivatives with longer maturities are less liquid, we also investigate a second case that involves rolling over the one-year-to-maturity variance product and the options on a yearly basis. Table 7 shows some promising results for this “rolling over” strategy. The position in variance product is modest, but the improvements along almost all criteria remain, and the structure of the optimal portfolio is similar to the one for the baseline case.¹⁰

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### Table 4: The Optimal Portfolios When Jump Risk Premium Is Set to Zero (i.e., $\lambda^0 = \lambda$)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Bonds</th>
<th>Stocks</th>
<th>Put</th>
<th>Collar</th>
<th>Variance</th>
<th>Call</th>
<th>CER</th>
<th>VaR</th>
<th>TVaR</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Prob R</th>
<th>Prob Infl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73.13%</td>
<td>26.87%</td>
<td></td>
<td></td>
<td></td>
<td>3.83%</td>
<td>−3.07%</td>
<td>−4.63%</td>
<td>4.6%</td>
<td>4.7%</td>
<td>3.8%</td>
<td>67%</td>
<td>46%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>77.38%</td>
<td>22.98%</td>
<td>−0.36%</td>
<td></td>
<td></td>
<td>3.84%</td>
<td>−3.40%</td>
<td>−5.40%</td>
<td>4.5%</td>
<td>4.8%</td>
<td>3.6%</td>
<td>70%</td>
<td>47%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>74.94%</td>
<td>25.09%</td>
<td>−0.02%</td>
<td></td>
<td></td>
<td>3.83%</td>
<td>−3.09%</td>
<td>−4.73%</td>
<td>4.6%</td>
<td>4.6%</td>
<td>3.8%</td>
<td>67%</td>
<td>46%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>79.42%</td>
<td>22.45%</td>
<td>−1.87%</td>
<td></td>
<td></td>
<td>3.85%</td>
<td>−3.29%</td>
<td>−5.12%</td>
<td>4.6%</td>
<td>4.9%</td>
<td>3.7%</td>
<td>69%</td>
<td>48%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>73.25%</td>
<td>27.15%</td>
<td>−0.17%</td>
<td></td>
<td></td>
<td>−0.23%</td>
<td>3.84%</td>
<td>−3.46%</td>
<td>−5.35%</td>
<td>4.5%</td>
<td>5.0%</td>
<td>3.5%</td>
<td>70%</td>
<td>50%</td>
</tr>
<tr>
<td>6</td>
<td>78.18%</td>
<td>23.69%</td>
<td>−0.06%</td>
<td></td>
<td></td>
<td>−1.66%</td>
<td>−0.15%</td>
<td>−3.45%</td>
<td>−5.38%</td>
<td>4.6%</td>
<td>5.0%</td>
<td>3.6%</td>
<td>71%</td>
<td>50%</td>
</tr>
</tbody>
</table>

**Table 4**: The Optimal Portfolios When Jump Risk Premium Is Set to Zero (i.e., $\lambda^0 = \lambda$)

**CE R =** certainty equivalent rate of return; **VaR =** value at risk; **TVaR =** tail value at risk; **SD =** standard deviation; **Prob R =** probability of earning more than the risk-free rate; **Prob Infl =** probability of earning more than the risk-free rate plus inflation.

### Table 5: The Optimal Portfolios When Volatility Risk Premium Is Set to Zero (i.e., $\zeta = 0$)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Bonds</th>
<th>Stocks</th>
<th>Put</th>
<th>Collar</th>
<th>Variance</th>
<th>Call</th>
<th>CER</th>
<th>VaR</th>
<th>TVaR</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Prob R</th>
<th>Prob Infl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.91%</td>
<td>44.09%</td>
<td></td>
<td></td>
<td></td>
<td>5.28%</td>
<td>−5.22%</td>
<td>−7.79%</td>
<td>7.3%</td>
<td>7.4%</td>
<td>6.3%</td>
<td>76%</td>
<td>65%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>60.13%</td>
<td>40.24%</td>
<td>−0.36%</td>
<td></td>
<td></td>
<td>5.29%</td>
<td>−5.18%</td>
<td>−8.11%</td>
<td>7.2%</td>
<td>7.4%</td>
<td>5.9%</td>
<td>78%</td>
<td>66%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>57.78%</td>
<td>42.24%</td>
<td>−0.02%</td>
<td></td>
<td></td>
<td>5.28%</td>
<td>−5.17%</td>
<td>−7.81%</td>
<td>7.3%</td>
<td>7.3%</td>
<td>6.3%</td>
<td>76%</td>
<td>65%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>69.02%</td>
<td>34.76%</td>
<td>−3.78%</td>
<td></td>
<td></td>
<td>5.40%</td>
<td>−5.23%</td>
<td>−8.17%</td>
<td>7.3%</td>
<td>7.8%</td>
<td>5.8%</td>
<td>78%</td>
<td>68%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>55.61%</td>
<td>44.81%</td>
<td>−0.14%</td>
<td></td>
<td></td>
<td>−0.28%</td>
<td>5.29%</td>
<td>−5.33%</td>
<td>−8.16%</td>
<td>7.2%</td>
<td>7.6%</td>
<td>5.8%</td>
<td>78%</td>
<td>67%</td>
</tr>
<tr>
<td>6</td>
<td>66.56%</td>
<td>37.25%</td>
<td>0.19%</td>
<td></td>
<td></td>
<td>−3.90%</td>
<td>−0.09%</td>
<td>5.40%</td>
<td>−5.31%</td>
<td>−8.13%</td>
<td>7.4%</td>
<td>7.9%</td>
<td>5.9%</td>
<td>78%</td>
</tr>
</tbody>
</table>

**Table 5**: The Optimal Portfolios When Volatility Risk Premium Is Set to Zero (i.e., $\zeta = 0$)

**CE R =** certainty equivalent rate of return; **VaR =** value at risk; **TVaR =** tail value at risk; **SD =** standard deviation; **Prob R =** probability of earning more than the risk-free rate; **Prob Infl =** probability of earning more than the risk-free rate plus inflation.
Practical Implications

The strategies discussed in the previous section yield superior results on the performance measures in our model. It is important, therefore, to consider to what extent the reported efficiency gains can also be captured when implemented for real-life pension plans. Implementation issues may arise because of market liquidity constraints and potential mismatches between existing portfolio structures and the structure of the proposed derivatives program.

Although most of the derivative products suggested above are frequently traded over the counter (OTC) as well as on exchanged-traded platforms, some may be substantially less liquidly available than their underlying equity portfolios.

For example, variance products are traded in OTC markets, but for larger pension plans (e.g., in excess of US$10 billion in assets), liquidity may not yet be sufficient to implement strategies for a meaningful notional amount, even under normal market conditions. Plain-vanilla equity derivatives (single or basket index puts and calls) are more liquidly available, although for large investors, transaction programs may take a substantial amount of time to complete, unless one is willing to accept significant implementation costs, yielding lower efficiency gains. Once derivative products become part of the overall strategy, investors should also manage liquidity risk in trading derivatives (i.e., the risk that the intended strategy cannot be maintained in the future when the derivative contracts expire). A potential mitigating strategy may be to diversify strategies in terms of their rollover horizon, so that

| Table 6: The Optimal Portfolios Following a Buy-and-Hold Strategy with a Four-Year Horizon |
|-----------------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Strategy | Bonds 51.22% | Stocks 48.78% | Put -1.20% | Collar -1.16% | Variance 24.25% | Call 2.50% | VaR -2.20% | TVaR 36.6% | Mean 34.9% | Median 19.9% | SD 91% | Prob R 79% |
| 2 | 58.45% | 42.75% | 4.09% | -1.56% | 35.0% | 33.5% | 17.6% | 92% | 80% |
| 3 | 39.21% | 61.95% | 2.03% | -2.65% | 35.9% | 34.6% | 18.7% | 91% | 81% |
| 4 | 103.91% | 27.97% | -31.9% | 6.82% | 0.79% | 37.4% | 37.4% | 15.6% | 95% | 87% |
| 5 | 49.05% | 53.01% | -0.39% | -1.67% | 24.40% | 3.66% | -1.98% | 34.6% | 33.4% | 16.8% | 92% | 82% |
| 6 | 98.70% | 34.93% | 1.18% | -34.2% | -0.60% | 27.72% | 5.86% | -0.06% | 38.7% | 38.8% | 16.8% | 94% | 87% |

**Table 7: The Optimal Portfolios Following a Rolling-Over Strategy with a Four-Year Horizon**

| Strategy | Bonds 52.21% | Stocks 47.79% | Put -0.15% | Collar 0.00% | Variance 24.13% | Call 2.75% | VaR -2.08% | TVaR 36.2% | Mean 34.5% | Median 19.5% | SD 91% | Prob R 79% |
| 2 | 53.35% | 46.80% | 3.07% | -1.89% | 35.8% | 34.2% | 19.1% | 91% | 79% |
| 3 | 52.20% | 47.80% | 2.75% | -2.08% | 36.2% | 34.5% | 19.5% | 91% | 79% |
| 4 | 61.77% | 40.99% | 5.25% | 0.24% | 35.1% | 33.9% | 17.1% | 93% | 81% |
| 5 | 53.29% | 46.88% | 3.08% | -1.89% | 35.8% | 34.2% | 19.1% | 91% | 79% |
| 6 | 60.78% | 41.87% | 0.12% | -2.77% | -0.01% | 25.22% | 4.99% | 0.06% | 35.4% | 34.1% | 17.4% | 93% | 81% |

**Table 6 and Table 7 Notes:**

CER = certainty equivalent rate of return; VaR = value at risk; TVaR = tail value at risk; SD = standard deviation; Prob R = probability of earning more than the risk-free rate; Prob Infl = probability of earning more than the risk-free rate plus inflation.
the entire notional amount of the strategy need not be rolled forward at a single time point.

It may also be cumbersome to align the derivative strategy with other parts of the existing portfolio. For example, basis risk may arise when equity investments are actively managed while the underlying value of the equity derivative relates to a market benchmark (e.g., the S&P 500 index). For internationally diversified equity portfolios, the benchmark may not be available in a single derivative contract, unless one is willing to trade OTC basket options tailored toward the investor’s specific equity portfolio. In addition, derivatives on internationally diversified portfolios may require different currency hedging than the underlying portfolio, which may be subject to intermediate portfolio rebalancing.

There may also be differences in treating call and put options. In the presence of active management, long positions in put options can be combined with an existing actively managed portfolio, if one is willing to accept the basis risk. However, solutions that require less investment in the underlying portfolio, replacing it with a position in long call options, cannot be combined with active management at all. Some strategies may therefore trigger transition or opportunity costs, which also need to be taken into account when evaluating derivatives strategies.

**Diversification across Risk Premia**

Having investigated the effect of including derivatives in the asset menu for a portfolio optimization problem faced by a pension fund, we find that when volatility risk premia and the possibility of sudden downward jumps in stock prices are taken into account, standard option strategies that use puts to protect the portfolio may be suboptimal under the volatility and jump risk premia reported in the literature. However, by writing instead of buying put options, or by exploiting the possibility of investing in the floating leg of variance swaps, marked improvements can be achieved in CER and in other performance measures more familiar to pension fund managers. The optimal strategies add value through diversification across risk premia. Sensitivity studies indicate that our conclusions remain valid under broad parameter ranges and even when the volatility risk premium or the jump risk premium is set to zero. The “rolling over” strategy for the derivatives, implemented as part of our robustness checks, shows further promising results.

More work remains to be done to facilitate the implementation of asset/liability management (ALM) strategies that focus on allocations in terms of risk premia instead of particular assets. However, we believe that such a focus will lead to better understanding and hence better control over exposure to different risks. Volatility-based derivatives are a good illustration of the possibilities in this direction, but we strongly believe that the more general principle of explicitly separating exposures to different risk drivers transcends that particular example.
Endnotes

1. The authors thank Scott Warlow, Patrick Savaria, Krishnan Chandrasekhar, Pierre Collin-Dufresne, Frank de Jong, Roderick Molenaar, Antoon Pelsser, Gabriel Petre, Krishnan Chandrasekhar, Berend Roorda, Jaap van Dam, and participants at the Rotman ICPM October 2011 Discussion Forum at the World Bank for their useful comments on earlier drafts of this article.

2. See, e.g., Bates (1996); Broadie, Chernov, and Johannes (2007); Broadie and Jain (2008); Duan and Yeh (2011); Eraker, Johannes, and Polson (2003); Liu and Pan (2003); Pan (2002); Todorov (2010); and Schürhoff and Ziegler (2011).

3. Redundancy of derivatives also critically depends on the possibility of trading continuously and in unrestricted quantities, and the optimal asset allocation may benefit from the inclusion of derivatives if this is no longer possible. This is also the case when one introduces transaction costs or restricts oneself to trading strategies that are not allowed to vary too quickly. Thus, it is clear that derivatives are not redundant in a realistic stochastic investment environment.

4. In reality, interest-rate risk must of course be mitigated separately (e.g., using interest-rate swaps), but we do not incorporate this factor in the present study.

5. This kernel and the formulas needed to price the derivative instruments are derived in a technical appendix that is available upon request. Vanilla call and put options on equity and the variance product can be priced explicitly in our model setup, as shown in the appendix.

6. The pension fund liabilities are deterministic in our model, given the constant interest rate assumption.

7. In the sensitivity analysis, we considered a longer horizon of four years.

8. In the model-free approach, information is extracted from high-frequency data of the underlying stocks and the model-free implied volatility data (e.g., VIX index).

9. Under the default parameter setting in our model, the average level of the implied volatility is 16.6%, and the average realized volatility in the stock market is about 13.6%.

10. The static rolling strategy illustrated here could be further extended to dynamics option strategies (see, e.g., Faias and Santa-Clara 2011).
References


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